Intermediary test

29.11.2018

Group A

1. Solve the problem

$$u''-(u')^2-u^2+u+1=0,\ u(0)=0.5,\quad u(\pi)=-0.5$$

using the finite difference method.

(compare your results using routine bvp4c from Matlab)

2. Solve numericaly using an efficient method

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \qquad 1<x<2, \quad 0<y<1;$$

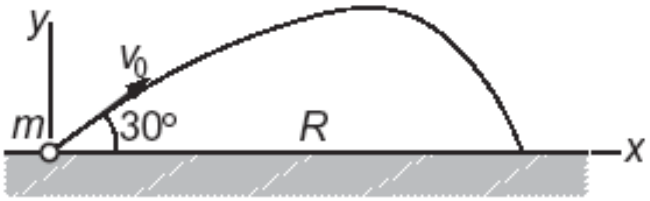
$$u(x,0)=2\ln x, \qquad u(x,1)=\ln(x^2+1), \quad 1\le x\le 2;$$

$$u(1,y)=\ln(y^2+1), \quad u(2,y)=\ln(y^2+4), \quad 0\le y\le 1.$$

and compare the results to the actual solution $u(x,y)=\ln(x^2+y^2)$.

Group B

1.



A ball of mass $m = 0.25$ kg is launched with the velocity $v_0 = 50$ m/s in the direction shown. Assuming that the aerodynamic drag force acting on the ball is $F_D = C_D v^{3/2}$, the differential equations describing the motion are

$$\ddot{x} = -\frac{C_D}{m}\dot{x}v^{1/2} \qquad \ddot{y} = -\frac{C_D}{m}\dot{y}v^{1/2} - g$$

where $v = \sqrt{\dot{x}^2 + \dot{y}^2}$. Determine the time of flight and the range R. Use $C_D = 0.03$ kg/(m·s)$^{1/2}$ and $g = 9.80665$ m/s^2.

(use Runge-Kutta method; compare the results with the ode45 routine from Matlab)

2.

Solve the initial boundary value problem for $u_t = u_{xx}$ on $-1 \le x \le 1$ for $0 \le t \le 0.5$ with initial data given by

$$u_0(x) = \begin{cases} 1 - |x| & \text{for } |x| < \frac{1}{2}, \\ \frac{1}{4} & \text{for } |x| = \frac{1}{2}, \\ 0 & \text{for } |x| > \frac{1}{2}. \end{cases}$$

Use the boundary conditions

$$u(t, -1) = u^*(t, -1) \quad \text{and} \quad u_x(t, 1) = 0,$$

where $u^*(t, x)$ is the exact solution given by

$$u^*(t, x) = \frac{3}{8} + \sum_{\ell=0}^{\infty} \left(\frac{(-1)^\ell}{\pi(2\ell + 1)} + \frac{2}{\pi^2(2\ell + 1)^2} \right) \cos \pi(2\ell + 1)x \, e^{-\pi^2(2\ell+1)^2 t}$$
$$+ \sum_{m=0}^{\infty} \frac{\cos 2\pi(2m + 1)x}{\pi^2(2m + 1)^2} e^{-4\pi^2(2m+1)^2 t}.$$